An Approach to Improve the Life Tesing Plans under Exponential – Poisson Distribution

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ABSTRACT

Statistical Quality Control is an important field in production and maintenance of quality product in manufacturing environments. Reliability sampling plans are widely used in manufacturing industries to monitor the quality of products in order to safeguard both the producer and consumer which simultaneously save the cost and time of an experiment. In this article, a new lifetime distribution named as Exponential – Poisson (EP) distribution is studied. The probability of acceptance for the single sampling is designed along with its associated decision rule are given to obtain the smallest sample size for the proposed two parameter probability distribution. In this study, the specified mean lifetime is calculated and the design parameter such as sample sizes, acceptance number are determined to study the desired quality levels such as Acceptable Reliability Level (ARL), Indifference Reliability Level (IRL) and Rejectable Reliability Level (RRL), its associated OC curve and the minimum ratio values are provided for the specified producer's risk. Table values are obtained and given for the easy selection of the plan parameters. Further, suitable illustration for the single sampling plan is given to study the plan parameters with a real time situations.

Keywords: Exponential – Poisson (EP) Distribution, Reliability, Single Sampling Plan.

1. Introduction

The pioneering methods of monitoring and action to control the acceptability or non-acceptability of a product and its service are carefully examined by statistical techniques known as Statistical Quality Control (SQC).

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Acceptance sampling is one of the methods that can be used to evaluate the quality of the product when it is lifetime then the sampling plan is called a Reliability sampling plan. In reliability different techniques are used to examine the quality of the manufactured items are modelled by the continuous probability distribution. When the test item is in go-no-go basic the sampling plan is attribute in nature which saves the experimenter cost and time. Suitable sampling plans are derived which aim to identify the plan parameters, such as the sample size and acceptance number.

In a reliability acceptance sampling, the lifetime of the test items or the number of failures observed throughout a predetermined testing period are examined, taking into account the risks to both the producer and the consumer, and a decision is made regarding whether to accept or reject the corresponding lot. Thus, this strategy took into account two different sorts of risks in which the consumer chooses to accept or reject a large number of items sent by the manufacturer based on the inspection. Setting a lower confidence limit on the mean life is the goal of these lifetime investigations. The scientific basis for such decision criteria is the total number of failures detected in the sample of size 'n' over the specified period of time 't'. If the observed failure rate is higher than 'c' (the acceptance number), the lot is rejected; otherwise, it is accepted. Several authors including Peach (1947), Cameron (1952), Epstein (1954), Hamaker (1958), Sobel and Tischendrof (1959), Dodge and H.G. Romig (1969), Schilling and John (1980), have researched the broad field of quality control and acceptance sampling plan focused on life testing.

In this article, it is assumed that the lifetime of the product follows Exponential Poisson distribution which was introduced by Kus (2007), Barreto-Souva (2013) proved that EP distribution is a better alternative to gamma distribution and Kaviyarasu and Fawaz (2017) developed Acceptance Sampling Plans for Percentiles Based on the Modified Weibull Distribution and Weibull-Poisson distribution. Based on these references, this article is developed for truncated acceptance sampling under Exponential Poisson distribution, and the values of minimum sample size, OC values and minimum ratio were presented. Finally, a numerical illustration is given with a real-time example for a better understanding of the plan parameters.

2. Exponential – Poisson distribution

In today's modern manufacturing industries lifetime distributions are widely used in a situation where failure occurs in a production process. When defectives are increasing exponentially, then the EP distribution is used to model the failure rate based on the distributional assumption. Combining the exponential distribution and the zero-truncated Poisson distributions leads to the discovery of the Exponential-Poisson (EP) distribution.

The probability density function of Exponential-Poisson distribution is

$$f(x;\theta) = \frac{\lambda\beta}{(1-e^{-\lambda})} e^{-\lambda-\beta x + \lambda \exp(-\beta x)}, \qquad x, \beta, \lambda \in R_+$$
(1)

The Cumulative distribution function of Exponential - Poisson distribution is

$$F(x;\theta) = (e^{\lambda \exp(-\beta x)} - e^{\lambda})(1 - e^{\lambda})^{-1}$$
(2)

Where $\theta = (\lambda, \beta), \lambda > 0$ is the shape parameter, $\beta > 0$ is the scale parameter of the Exponential distribution and λ is the Poisson parameter. When $\lambda \rightarrow 0$, Exponential Poisson distribution reduces Exponential distribution with parameter β .

3. Design of the plan

In this research article, a single sampling plan under the presumption of the lifetime distribution follows Exponential-Poisson (EP) distribution.

In a truncated life testing, an acceptance sampling plan comprises the following components:

- 1. The number of units 'n'
- 2. The lot is accepted if there are no more than c failures out of n detected at the end of the predetermined time t, which is represented by the acceptance number c.
- 3. The ratio t/β_0 , where β_0 is known as the specified mean life.

4. Minimum Sample Size

Consumer risk is the probability of not rejecting a bad lot (i.e., accepting a good lot) and in this study, it is settled to not exceed 1-P*. The actual average life is smaller than the specified life is β_0 . By the implication of a binomial distribution, the acceptance criteria of a lot can be determined, and it is necessary to consider that the size of the lot is adequately large and assumed that it should be infinite.

To identify the required smallest sample size (n), the given inequality must be

satisfied.
$$\sum_{i=1}^{c} {n \choose i} p^{i} (1-P)^{n-i} \le 1-P^{*}$$
 (3)

The minimal sample sizes that satisfy the inequality specified above are

 $t/\beta_0 = 0.25, 0.50, 0.75, 1, 1.25, 1.5, 1.75, 2, 2.25, 2.5, 2.75, 3, 3.25, 3.5, 3.75, 4$ with P* = 0.75, 0.90, 0.95, 0.99 and C = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. The results are shown in Table-2.

5. Operating Characteristic of Sampling Plan

The Operating Characteristics (OC) of the SSP $(n, c, t/\beta_0)$ gives the decision criteria of accepting a good lot.

$$L(p) = P \text{ (Accepting a lot)}$$
$$L(p) = \sum_{i=1}^{c} {n \choose i} p^{i} (1-P)^{n-i}$$
(4)

Here $p = F(t; \theta)$ is the parameter of the lot quality (i.e. function of θ). The product's average lifetime is increasing for the fixed time 't' $\beta > \beta_0$ indicating that the failure probability $p = F(t; \theta)$ which is a monotonically decreasing function $\beta > \beta_0$, and thus, the average lifetime of OC function is increasing; which implies that the ratio β/β_0 value is increasing say from "0.25 to 2. Table-3 presents OC values as a function of $\beta > \beta_0$ for the single sampling plan $(n, c, t/\beta_0)$.

6. Producer Risk

The probability that the producer will reject the lot when $\beta > \beta_0$. One may be willing to figure out the value of β / β_0 which will be guaranteed that the producer's risk does not exceed or equal 0.05, suppose the considered sampling plan is implemented on the basis of the studied single sampling plan and also a certain producer's risk value that is 0.05. The value of β / β_0 is the least nonnegative integer, $p = F\left(\frac{t}{\beta_0} \frac{\beta_0}{\beta}\right)$ that satisfies the given inequality. $P_r(p) = P_r$ (Rejecting the lot)

$$\sum_{i=1}^{c} \binom{n}{i} p^{i} (1-P)^{n-i} \ge 0.95$$
(5)

The smallest values of β / β_0 satisfy the given inequality (5) for the proposed plan $(n, c, t/\beta_0)$ at a specific confidence level P*.

7. Basic Reliability Quality Levels

In this article, the proposed sampling plan is called as Single Sampling Plan (SSP) indexed through Incoming and Outgoing reliability quality levels performed measures are indexed through Acceptable Reliability Level (ARL), Limiting Reliability Level (LRL) and Indifference Reliability Level (IRL). Thus, the parameters $(n, c, t/\beta_0)$ are used to characterize the sampling plan.

8. Real time example

In recent days, the number of people suffering from high blood pressure has been rising gradually. High blood pressure can cause severe health disabilities like blindness, memory loss, heart attacks, and other related problems; therefore, it is essential to monitor blood pressure to prevent these problems using proper medical instruments. The medical electronic device known as a digital sphygmomanometer is used to monitor and measure the patient's pulse rate to check their blood pressure are under control or not. The digital sphygmomanometer includes a gauge, stethoscope, and error indicator in one device, and the blood pressure reading gets displayed on a small screen. It is a sensor-based device, and it is simple to operate anyone at the home without any assistance. Due to the rising number of BP patients, the demand for digital sphygmomanometers is highly increased. Hence the manufacturer produces large number of the medical instruments and distributed all over the parts which decided to increase the production of this digital monitor.

On this occasion, the production of digital sphygmomanometers is significantly increasing, it follows the exponential process. In this production process, the probability of occurrence of defectives follows the Poisson process i.e., the time between product failures follows a Poisson distribution. As a result, Exponential Poisson (EP) is an acceptable and appropriate probability distribution for this case.

9. Numerical Illustration

1

1.5

2

Consider that the overall life of an item (Sphygmomanometer) which follows an EP distribution with the parameters $\lambda=2$. The quality inspector desires to investigate the item's average lifetime of 1000 hours at the confidence level which is P*= 0.75. The test was terminated after 500 hours. This will lead to the ratio $t/\beta_0 = \frac{500}{1000} = 0.5$, from the Table-2 the results for the minimum sample size are obtained as for c=2. The sampling plan which is used by the experimenter is (n=22, c=5, $t/\beta_0 = 0.50$).



Table-1: OC values for (n=22, c=5, t/β_0 =0.5) under EP for p* = 0.75

3

3.5

4

4.5

5

2.5

Figure-1: OC curve for the plan (22, 5, 0.5) under EP for $p^* = 0.75$.

10. Conclusion

In this paper, a new lifetime probability distribution is studied under a single-acceptance sampling plan when the lifetime of an item follows the Exponential Poisson distribution. This provides a higher probability of acceptance when the mean lifetime is predefined as a quality parameter since EP Distribution is a useful model in the areas of quality control and reliability studies. The procedures and tables for testing the quality of life of an item are evaluated in terms of reliability testing indexed through ARL, IRL, and LRL. The tables

p

are exhibited for the minimum sample size required to guarantee a certain mean lifetime of the test items. The operating characteristic values and the associated producer's risks are also discussed and displayed in Table - 3 and Table - 4. The table values are explained with suitable real-life example and illustration.

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Appendix

TABLE-2: Minimum sample sizes for the EP distribution.

		t/β_0									
P*	n	0.25	0.5	0.75	1	1.5	2	2.5	3	3.5	4
0.75	0	3	2	1	1	1	1	1	1	1	1
0.75	1	6	5	5	5	4	3	2	2	2	2
0.75	2	9	6	6	6	6	5	5	4	4	4
0.75	3	12	9	8	8	8	7	5	5	5	5
0.75	4	14	11	11	11	8	8	6	6	6	6
0.75	5	24	22	12	12	10	7	7	7	7	7
0.75	6	26	23	14	12	12	10	10	8	8	8
0.75	7	29	28	20	16	12	12	10	9	9	9
0.75	8	35	33	20	16	10	10	10	10	10	10
0.75	9	41	38	20	16	13	12	11	11	11	11
0.75	10	47	44	22	18	14	13	12	12	12	12
0.9	0	5	4	4	3	3	3	2	2	1	1
0.9	1	8	8	8	6	5	4	3	3	3	3
0.9	2	12	10	10	7	6	4	4	4	4	4
0.9	3	15	12	12	10	8	6	6	5	5	5
0.9	4	18	16	12	12	8	8	6	6	6	6
0.9	5	21	18	12	12	10	8	8	8	7	7
0.9	6	23	18	16	12	12	10	10	8	8	8
0.9	7	26	23	16	13	13	12	12	9	9	9
0.9	8	29	20	20	15	13	12	12	11	11	10
0.9	9	32	29	20	16	13	12	12	11	11	11
0.9	10	35	33	23	18	14	13	12	12	12	12
0.95	0	6	5	5	5	3	3	2	2	1	1
0.95	1	10	7	5	5	5	3	3	3	3	3
0.95	2	13	9	6	6	6	5	5	4	4	3
0.95	3	17	13	13	8	8	6	6	5	5	5
0.95	4	20	15	14	10	8	8	7	7	6	6
0.95	5	23	19	15	12	10	8	8	8	7	7
0.95	6	26	19	16	13	10	10	10	8	8	8
0.95	7	29	23	16	13	13	10	10	10	10	9
0.95	8	32	28	20	16	14	12	12	10	10	10
0.95	9	35	31	22	18	15	13	13	11	11	11
0.95	10	38	33	23	18	16	15	13	13	13	12
0.99	0	9	6	6	4	3	3	2	2	1	1
0.99	1	13	7	7	7	5	4	3	3	3	3
0.99	2	17	12	9	9	6	5	5	4	4	4
0.99	3	21	15	9	9	6	6	6	5	5	5
0.99	4	24	15	10	9	9	8	7	7	6	6
0.99	5	28	19	12	10	10	8	8	8	7	7
0.99	6	31	20	12	12	12	10	10	9	8	8
0.99	7	34	23	16	13	13	12	10	10	10	9
0.99	8	37	28	22	14	14	12	12	11	11	10
0.99	9	41	29	23	18	16	14	12	12	12	11
0.99	10	44	33	25	20	16	14	14	12	13	12

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			β / β_0								
P*	N	t/β_0	1	1.5	2	2.5	3	3.5	4	4.5	5
0.75	24	0.25	0.2283	0.5762	0.7886	0.8934	0.9437	0.9691	0.9822	0.9893	0.9933
0.75	22	0.5	0.1105	0.3859	0.629	0.7835	0.8728	0.9235	0.9528	0.97	0.9804
0.75	12	0.75	0.013	0.1105	0.2898	0.4765	0.629	0.7482	0.819	0.8728	0.9097
0.75	12	1	0.0015	0.0269	0.1105	0.2416	0.3859	0.5185	0.629	0.7164	0.7835
0.75	10	1.5	0.0009	0.1686	0.7362	0.1727	0.2939	0.4164	0.5274	0.6219	0.6993
0.75	7	2	0.0422	0.1715	0.34	0.4986	0.6278	0.7262	0.7986	0.8513	0.8894
0.9	21	0.25	0.0764	0.3414	0.5996	0.7677	0.8648	0.9195	0.9507	0.969	0.9799
0.9	18	0.5	0.0026	0.0477	0.177	0.3494	0.5129	0.645	0.7438	0.8152	0.866
0.9	12	0.75	0.013	0.1105	0.2898	0.4765	0.629	0.7408	0.819	0.8728	0.9097
0.9	12	1	0.0015	0.0269	0.1105	0.2416	0.3859	0.5185	0.629	0.7164	0.7835
0.9	10	1.5	0.0009	0.0169	0.7362	0.1727	0.2939	0.4164	0.5274	0.6219	0.6993
0.9	8	2	0.0054	0.0472	0.1414	0.2675	0.3986	0.518	0.6189	0.7006	0.7652
0.95	23	0.25	0.0413	0.2495	0.5034	0.6928	0.8124	0.8841	0.927	0.9529	0.9689
0.95	19	0.5	0.0013	0.0315	0.1354	0.2915	0.4579	0.5894	0.6968	0.7771	0.8357
0.95	15	0.75	0.0007	0.019	0.0919	0.2175	0.3623	0.4984	0.6132	0.7045	0.7748
0.95	12	1	0.0015	0.0269	0.1105	0.2416	0.3859	0.5185	0.629	0.7164	0.7835
0.95	10	1.5	0.0009	0.0169	0.0736	0.1727	0.2939	0.4164	0.5274	0.6219	0.6993
0.95	8	2	0.0054	0.0472	0.1414	0.2675	0.3986	0.518	0.6189	0.7006	0.7652
0.99	28	0.25	0.0077	0.1009	0.2956	0.4982	0.6578	0.7696	0.8445	0.894	0.9267
0.99	19	0.5	0.0013	0.0315	0.1354	0.2915	0.4519	0.5894	0.6968	0.7771	0.8357
0.99	12	0.75	0.013	0.1105	0.2898	0.4765	0.629	0.7408	0.819	0.8728	0.9097
0.99	10	1	0.0169	0.1189	0.2939	0.4738	0.6219	0.7321	0.8104	0.865	0.9031
0.99	10	1.5	0.0009	0.0169	0.0736	0.1727	0.2939	0.4163	0.5274	0.6219	0.6993
0.99	8	2	0.0054	0.0472	0.1414	0.2675	0.3986	0.518	0.6189	0.7006	0.7652

TABLE-3: OC values for (n, c =5, $t/\beta_0 = 0.50$) for a given P* under EP distribution.

		t/β_0										
P*	c	0.25	0.5	0.75	1	1.5	2	2.5	3	3.5	4	
0.75	0	33.522	44.318	33.114	44.1593	66.4326	89.244	111.46	133.29	154.57	175.84	
0.75	1	3.9971	6.5269	9.8241	13.0986	15.4181	14.834	11.204	13.448	15.699	17.95	
0.75	2	2.5772	3.2854	4.9307	6.57419	9.80517	10.576	13.191	11.08	12.087	13.092	
0.75	3	2.0401	2.9708	3.9003	5.20038	6.50061	7.7957	7.7685	8.9041	6.5701	7.3001	
0.75	4	2.1384	3.7801	4.1404	5.52056	4.33612	5.2027	6.0446	6.9082	7.7576	5.9897	
0.75	5	1.5036	2.0414	3.0558	4.07441	5.08414	4.9453	5.7684	4.1738	4.679	5.2196	
0.75	6	1.3382	1.9023	2.3747	3.16627	3.95779	3.8045	4.4385	5.0794	5.7136	4.7245	
0.75	7	1.279	2.3021	3.4437	3.58653	3.19488	3.8425	4.4829	5.1302	5.7707	5.0578	
0.75	8	1.2394	1.9349	2.9019	2.99219	2.64534	2.4751	2.8917	3.3048	3.7179	4.131	
0.75	9	1.1977	1.6564	2.4845	2.55859	2.47438	2.9648	3.1137	3.5585	3.5469	3.941	
0.75	10	1.1329	1.6025	2.4037	2.55022	2.3397	2.8076	2.9721	3.3967	3.4066	3.7852	
0.9	0	24.026	38.824	58.757	58.4195	87.3278	117.68	98.23	116.56	69.764	79.126	
0.9	1	5.3956	10.845	16.316	15.9565	19.6502	20.523	18.555	22.227	25.969	29.688	
0.9	2	3.4986	5.7448	8.6195	7.77345	9.86147	8.0389	10.018	11.016	12.088	13.027	
0.9	3	2.5952	4.0729	6.1092	6.67171	6.49225	7.8006	6.4634	7.3922	8.3162	9.2371	
0.9	4	2.4067	3.5314	4.9215	4.50069	5.62319	5.2033	6.059	6.9249	7.7905	7.3225	
0.9	5	1.8863	3.2109	3.0517	4.0941	5.11807	4.9453	5.769	5.0156	5.6425	6.263	
0.9	6	1.6405	2.5096	3.3012	3.16638	3.95798	4.742	4.4386	5.0835	5.7189	6.354	
0.9	7	1.5346	2.6797	2.6913	2.81808	3.5226	4.2204	3.5501	4.0573	4.5644	5.0716	
0.9	8	1.4498	1.9334	2.8933	2.78194	2.92569	3.5121	3.7008	4.2292	4.7579	5.2865	
0.9	9	1.3868	2.4931	2.4909	2.55966	2.47779	2.9735	3.1139	3.5587	4.0036	4.4484	
0.9	10	1.3353	2.5068	2.532	2.55024	2.33976	2.8077	2.9722	3.3968	3.4016	3.7796	

TABLE-4: Minimum ratio of true mean life over β_0 at the producer's risk of 0.05

0.95	0	29.304	48.902	72.763	97.0202	87.3297	117.68	98.228	117.87	70.31	79.741
0.95	1	6.7455	9.3748	9.7967	13.0623	19.6502	14.757	18.555	22.266	25.96	29.678
0.95	2	3.7585	6.9758	4.9285	6.57138	9.86147	10.579	13.22	14.542	12.022	12.985
0.95	3	2.9617	4.4461	6.6371	5.20012	6.50052	7.8006	6.43	7.3718	8.2934	9.2149
0.95	4	2.9101	3.5418	3.3734	3.97527	4.96909	5.9629	6.9505	6.9235	7.7889	7.3229
0.95	5	2.0833	3.3776	3.9236	4.07288	5.0911	4.9469	5.771	5.0157	5.6427	6.2631
0.95	6	1.8729	2.9614	3.3097	3.49201	4.36501	3.8045	4.4386	5.0659	5.6991	6.354
0.95	7	1.7208	2.6838	2.6911	2.81602	3.52003	4.2259	3.5501	4.0571	4.5642	5.0716
0.95	8	1.6129	2.7834	2.9	2.99075	3.73844	3.8439	4.4813	4.2292	4.7578	5.2865
0.95	9	1.5269	2.6801	2.7668	2.93437	3.43682	3.5523	4.1442	3.9589	4.4538	4.9486
0.95	10	1.458	2.5142	2.535	2.55356	3.18925	3.3223	3.876	4.087	4.5979	4.2424
0.99	0	43.622	57.727	87.445	78.3402	87.3287	117.67	98.226	118.1	70.31	79.288
0.99	1	8.9589	9.4444	14.184	18.5538	19.6502	20.524	18.555	22.275	25.96	29.686
0.99	2	4.9731	6.9731	7.7033	10.2461	9.86147	10.55	13.22	11.024	12.032	13.039
0.99	3	3.6998	5.1916	4.4493	5.93145	7.42576	5.507	6.464	7.3724	8.2937	9.2149
0.99	4	2.9234	3.5308	3.3696	3.98518	4.98254	5.9572	6.9526	6.9235	7.789	7.3229
0.99	5	2.5599	3.3797	3.0582	3.29843	4.1231	4.9466	5.7711	5.0157	5.6427	6.2631
0.99	6	2.2408	2.8218	2.3747	3.16103	3.95127	4.7418	4.4386	5.0659	5.6991	6.354
0.99	7	2.0359	2.6852	2.6899	2.81331	3.51663	4.2202	4.483	5.1165	4.5642	5.0716
0.99	8	1.8776	2.7837	3.2212	2.56274	3.20342	3.8441	4.483	4.2296	4.7578	5.2865
0.99	9	1.8029	2.4997	2.9048	2.93095	3.20342	3.8441	3.8084	4.3404	4.8829	4.4484
0.99	10	1.7038	2.4997	2.7813	2.88121	3.18781	3.3297	3.8728	3.7443	4.2124	4.6731